Period 3, April 23, 2025

If $x+3y^{\frac{1}{3}}=y$, what is $\frac{dy}{dx}$ at the point $(2,8)$?

$X+3y^{\frac{1}{3}}=y$

$1+3\cdot\frac{1}{3}\cdot y^{\frac{1}{3}-1}\frac{dy}{dx}=\frac{dy}{dx}$

$1+y^{-\frac{2}{3}}\frac{dy}{dx}=\frac{dy}{dx}$

$1+\frac{1}{\sqrt[3]{y^2}}\frac{dy}{dx}=\frac{dy}{dx}$

$1+\frac{1}{\sqrt[3]{8^2}}\frac{dy}{dx}=\frac{dy}{dx}$

$1+\frac{1}{\sqrt[3]{64}}\frac{dy}{dx}=\frac{dy}{dx}$

$1+\cancel{\frac{1}{4}\frac{dy}{dx}}=1\frac{dy}{dx}-\frac{1}{4}\frac{dy}{dx}$

$\cancel{-\frac{1}{4}\frac{dy}{dx}}$

$\frac{4}{3}\cdot 1=\cancel{\frac{4}{3}}\cancel{\frac{3}{4}}\frac{dy}{dx}$

$\frac{4}{3}=\frac{dy}{dx}$

(A) $\frac{1}{3}$

(B) $\frac{3}{4}$

(C) $\frac{5}{4}$

(D) $\frac{4}{3}$

$\lim_{x\to\infty}\frac{10-6x^2}{5+3e^x}$ is $\Rightarrow \lim_{x\to\infty}\frac{0-12x}{0+3e^x}\Rightarrow\lim_{x\to\infty}\frac{-12}{3e^x}=\frac{-12}{\infty}=0$

$\frac{\infty}{\infty}$ $\frac{\infty}{\infty}$

(A) -2

(B) 0

(C) 2

(D) nonexistent

$$f(x)=\begin{cases}-x^2+3 & \text{if } x\le 5\\ -10x+28 & \text{if } x>5\end{cases}$$

Continias

$-(5)^2+3=-25+3=-22$

$-10(5)+28=-50+28=-22$ } continous

$-2x=-2(5)=-10$

$-10=-10=-10$ } diff

Let f be the function defined above.

(A) f is continuous and differentiable at $x=5$.

(B) f is continuous but not differentiable at $x=5$.

(C) ~~f is differentiable but not continuous at $x=5$.~~

(D) f is defined but neither continuous nor differentiable at $x=5$.

If $\frac{dy}{dx}=2-y$, and if $y=1$ when $x=1$, then $y=$

$\frac{dx}{(2-y)}\cdot\frac{dy}{dx}=(2-y)\cdot\frac{dx}{(2-y)}$

(A) $2-e^{x-1}$

$\int\frac{1}{2-y}\,dy=\int 1\,dx$

(B) $2-e^{1-x}$

$u=2-y$

$du=-dy$

$-du=dx$

$-\ln|2-y|=x+c$

$-\ln|2-1|=1+C$

$-\ln 1=1+C$

(C) $2-e^{-x}$

$-0=1+C$

$-1=C$

(D) $2+e^{-x}$

$-1\cdot-\ln|2-y|=(x-1)\cdot-1$

$e^{\ln|2-y|}=-x+1$

$2-y=e^{-x+1}$

$-2 \qquad -2$

$-1\cdot -y=(-2+e^{-x+1})\cdot -1$

$\Rightarrow y=2-e^{1-x}$

$\int \frac{x^2+1}{\left(x^3+3x-5\right)^3} dx =$

$\int \frac{x^2+1}{(x^3+3x-5)^3}$

$u = x^3+3x-5$

$du = (3x^2+3)\,dx$

$\frac{du}{3x^2+3} = dx$

$\int \frac{\cancel{x^2+1}}{u^3} \cdot \frac{du}{3\cancel{(x^2+1)}} = \frac{1}{3}\int u^{-3}\,du$

$-3+1=-2$

$\frac{1}{3} \cdot \frac{1}{-2} \cdot u + C$

$-\frac{1}{6} \cdot \frac{1}{u^2} + C$

$-\frac{1}{6} \cdot \frac{1}{(x^3+3x-5)^2}$

(A) $-\frac{3}{2} \cdot \frac{1}{\left(3x^2+3\right)^2} + C$

(B) $-\frac{1}{6} \cdot \frac{1}{\left(3x^2+3\right)^2} + C$

(C) $-\frac{3}{2} \cdot \frac{1}{\left(x^3+3x-5\right)^2} + C$

(D) $-\frac{1}{6} \cdot \frac{1}{\left(x^3+3x-5\right)^2} + C$

In which of the following intervals must there be a number c such that $f'(c) = 2$?

x	0	4	8	12	16
$f(x)$	8	0	2	10	1

The table above gives selected values for the differentiable function f.

(A) $(0, 4)$ $\frac{0-8}{4-0} = \frac{-8}{4} = -2$

(B) $(4, 8)$ $\frac{2-0}{8-4} = \frac{2}{4} = \frac{1}{2}$

(C) $(8, 12)$ $\frac{10-2}{12-8} = \frac{8}{4} = 2$

(D) $(12, 16)$ $1-10 = -\frac{9}{4} \neq 2$

$\overline{16}$-12 4

Which of the following is an expression for the position of the particle at time $t \geq 0$?

A particle moves along a straight line so that at time $t \geq 0$ its acceleration is given by $a(t) = 12t$. At time $t = 0$, the velocity of the particle is 2 and the position of the particle is 5.

Ⓐ ~~$6t^2 + 5$~~

Ⓑ ~~$6t^3 + 2t + 5$~~

Ⓒ ~~$2t^3 + 5$~~

Ⓓ $2t^3 + 2t + 5$

$a(T) = 12T$

$V(T) = \int a(T)\,dT = \int 12T\,dT = 6T^2 + C$

$V(T) = 6T^2 + C$

$2 = 6(0)^2 + C$

$2 = C$

$S(T) = \int v(T)\,dT = \int (6T^2 + 2)\,dT = 2T^3 + 2T + C$

$5 = 2(0)^3 + 2(0) + C$

$5 = 0 + 0 + C$

$5 = C$

$2T^3 + 2T + 5$

Let g be the function given by $g(x) = \int_3^x \left(t^2 - 5t - 14\right) dt$. What is the x-coordinate of the point of inflection of the graph of g?

$g'(x) = x^2 - 5x - 14$

$g''(x) = 2x - 5$

$g''(x) = 0$ or $\emptyset$

(A) -2

(B) $\frac{5}{2}$

$0 = 2x - 5$

$5 = 2x$

$\frac{5}{2} = x$

(C) 3

(D) 7

A particle moves along the y-axis so that at time $t \geq 0$ its position is given by $y(t) = t^3 - 4t^2 + 4t + 3$. Which of the following statements describes the motion of the particle at time $t = 1$?

$V(T) = 3T^2 - 8T + 4$

$a(T) = 6T - 8$

(A) The particle is moving down the y-axis with decreasing velocity. ✓ 4

$V(T)$

(B) The particle is moving down the y-axis with increasing velocity. 2

$V(1) = 3(1)^2 - 8(1) + 4 = -1$ ← Moving Down

$a(1) = 6(1) - 8 = -2$

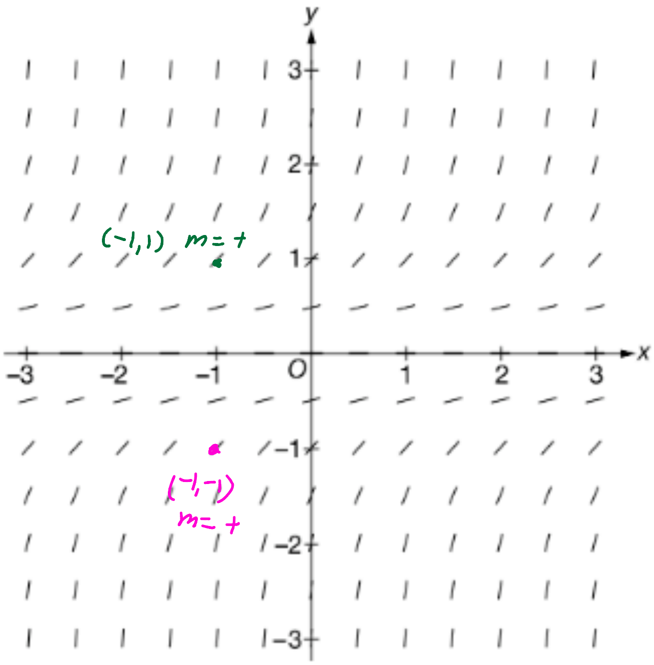
(C) The particle is moving up the y-axis with decreasing velocity. 3

$V(T)$ is decreasing

(D) The particle is moving up the y-axis with increasing velocity. 1

Speed is increasing $V(T) = -$

$a(T) = -$



(A) $\cancel{\frac{dy}{dx} = |x+y|}$ $(-1,1)$ $|-1+1| = 0$ m=+ $(-1,1)$

(B) $\cancel{\frac{dy}{dx} = x^3}$ $(-1)^3 = -1$

(C) $\frac{dy}{dx} = y^3$ $(1)^3 = +1$ $\cancel{(-1)^3 = -1}$

(D) $\frac{dy}{dx} = y^2$ $\underbrace{(1)^2 = 1 \qquad (-1)^2 = 1}$

The equation $y = 2e^{6x} - 5$ is a particular solution to which of the following differential equations?

$\underbrace{y+5}= 2e^{6x}$

Ⓐ $y' - 6y - 30 = 0$

Ⓑ $2y' - 12y + 5 = 0$

Ⓒ $y'' - 5y' - 6y = 0$

Ⓓ $y'' - 2y' + y + 5 = 0$

$y = 2e^{6x} - 5$

$y' = 2 \cdot e^{6x} \cdot 6 + 0$

$y' = 2e^{6x} \cdot 6 \Rightarrow y' = (y+5) \cdot 6$

$y' = 6y + 30$

$-6y \quad -6y \quad -30$

-30

$y' - 6y - 30 = 0$

What is the rate of change, in radians per second, of angle θ at the instant when the balloon is 40 feet above point P ?

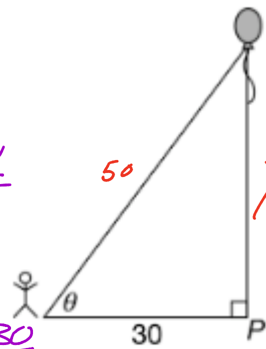
$\tan\theta = \frac{y}{30}$

$30\tan\theta = y$

$30 \cdot \sec^2\theta \cdot \frac{d\theta}{dT} = \frac{dy}{dT}$

$30 \cdot \left(\frac{50}{30}\right)^2 \frac{d\theta}{dT} = 2$

$\frac{\cancel{30}}{2500} \cdot \frac{2500}{\cancel{30}} \cdot \frac{d\theta}{dT} = 2 \cdot \frac{30}{2500}$



$\frac{dy}{dT} = 2 \text{ FT/sec}$

$y = 40$

$\sec = \frac{1}{\cos}$

$\frac{1}{\frac{30}{50}} = \frac{50}{30}$

A person stands 30 feet from point P and watches a balloon rise vertically from the point, as shown in the figure above. The balloon is rising at a constant rate of 2 feet per second.

$\frac{d\theta}{dT} = \frac{2 \cdot 30}{2500} = \frac{6}{250} = \frac{3 \cdot \cancel{2}}{\cancel{2} \cdot 125}$

(A) $\frac{3}{100}$

(B) $\frac{3}{125}$

(C) $\frac{1}{12}$

(D) $\frac{5}{27}$